Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 3 – statistics and probability

Wednesday 15 May 2019 (morning)

1 hour

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[50 marks]**.

2219 – 7210

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 16]

The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} kx & 0 \le x < 1 \\ kx^2 & 1 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}.$$

(a) Show that $k = \frac{6}{17}$. [4]

(b) Find the cumulative distribution function of X. [6]

(c) Find the median, m, of X. [3]

(d) Find $\mathrm{P}(|X - m| < 0.75)$. [3]

2. [Maximum mark: 12]

Employees answer the telephone in a customer relations department. The time taken for an employee to deal with a customer is a random variable which can be modelled by a normal distribution with mean 150 seconds and standard deviation 45 seconds.

(a) Find the probability that the time taken for a randomly chosen customer to be dealt with by an employee is greater than 180 seconds. [2]

(b) Find the probability that the time taken by an employee to deal with a queue of three customers is less than nine minutes. [4]

At the start of the day, one employee, Amanda, has a queue of four customers. A second employee, Brian, has a queue of three customers. You may assume they work independently.

(c) Find the probability that Amanda's queue will be dealt with before Brian's queue. [6]

3. [Maximum mark: 10]

In a large population of hens, the weight of a hen is normally distributed with mean μ kg and standard deviation σ kg. A random sample of 100 hens is taken from the population. The mean weight for the sample is denoted by $\bar{X}$.

(a) State the distribution of $\bar{X}$ giving its mean and variance. [1]

The sample values are summarized by $\sum x = 199.8$ and $\sum x^2 = 407.8$ where x kg is the weight of a hen.

(b) Find an unbiased estimate for μ. [1]

(c) Find an unbiased estimate for σ^2. [2]

(d) Find a 90 % confidence interval for μ. [3]

(e) It is found that $\sigma = 0.27$. It is decided to test, at the 1 % level of significance, the null hypothesis $\mu = 1.95$ against the alternative hypothesis $\mu > 1.95$.

(i) Find the p-value for the test.

(ii) Write down the conclusion reached. [3]

4. [Maximum mark: 12]

It is given that X, Y, Z are random variables and c is a constant.

(a) Show that $\text{Cov}(X + c, Y) = \text{Cov}(X, Y)$. [3]

(b) Show that $\text{Cov}(X + Y, Z) = \text{Cov}(X, Z) + \text{Cov}(Y, Z)$. [3]

It is given that S and T are two independent normal variables with mean 0 and variance 1.

(c) Using the results from (a) and (b), find the value of $\text{Cov}\left(1 + S, S + ST^2\right)$. [6]